

**DELHI UNIV. B.COM.(HONS.)/IV SEM./2017
PAPER BCH 4.2 - BUSINESS MATHEMATICS**

Time : 3 hours

Maximum Marks : 75

Attempt All questions. Marks are indicated against each question.

1. (a) A two industry input-output relationships are given below:

	Consumption		Final Demand	Gross Output
	I	II		
I	16	20	4	40
II	8	40	32	80
Labour day	80	120		

Using matrix notations, determine :

- (i) Gross output required to satisfy the new final demand of 18 units and 44 units for industry I and II respectively.
- (ii) Total Labour days required.
- (iii) Equilibrium prices if wage rate is 40 per Labour day. (8)

OR

a) A firm produces three products P_1 , P_2 and P_3 processed on three machines M_1 , M_2 and M_3 before completion. M_1 can process 25 units of P_1 or 50 units of P_2 or 75 units of P_3 per hour. M_2 can process 50 units of any one product per hour. M_3 can process 50 or 25 or 100 units per hour of products P_1 , P_2 and P_3 respectively.

The processing hours available on machines M_1 , M_2 and M_3 are 12, 12 and 13 respectively.

Using matrix notations, find :

- (i) How many units of three products can be produced per day ?
 - (ii) The production cost per unit, if costs per hour of operating machines M_1 , M_2 and M_3 are ₹500, 1000 and ₹ 1500 respectively. (8)
 - (iii) The total cost of production.
- (b) Mr. X has invested ₹ 35000, that he had divided into three investments. Part of the money is invested in a savings account with an annual return of 6%, partly in 7% annual yield bonds and the remainder in business. In 2015, when he lost 6% of the money that he invested in the business, his net income from all the three investments is ₹ 660. If he invested ₹ 3,000 more in the business than in the savings account, how much was invested in each? Use matrix algebra. (4)

OR

A manufacturer produces three products X, Y and Z which he sells in three markets I, II, III. Monthly sale and sale prices are given as follows:

Market	Items Sold			Sale Price (in ₹)		
	X	Y	Z	X	Y	Z
I	1000	1500	2000	2.00	3.00	4.00
II	2000	2800	3000	2.50	2.80	3.70
III	3000	3500	4000	2.30	3.10	4.20

If the cost per item for three products X, Y, and Z are 1.80, ₹ 2.50 and ₹ 3.40 respectively, find by matrix algebra his total profit (4)

2. (a) A firm manufacturing office furniture provides the following information regarding resource consumption and availability and profit contribution:

Resources	Usage per unit			Availability
	Tables	Chairs	Book case	
Timber (cu. ft.)	8	4	3	640
Assembly deptt. (man hours)	4	6	2	540
Finishing deptt. (man hours)	1	1	1	100
Profit contribution per unit (₹)	30	20	12	

The firm wants to determine its optimal product mix:

- (i) Formulate the LPP and solve with the simplex method.
- (ii) Find the optimal product mix and the total maximum profit contribution.
- (iii) What are the shadow prices of resources? (12)

OR

A firm produces three products, A, B, C using three resources (material, machine hours and labour hours) The manager of the firm wants to find out the best production strategy. By using Simplex technique, the following solution was obtained.

$C_j \rightarrow$	30	40	10	0	0	0	
BV	X_1	X_2	X_3	S_1	S_2	S_3	Quantity
S_1	1/2	1	3/2	1/2	0	0	45
S_2	3/2	0	-1/2	-1/2	1	0	15
S_3	5/2	0	1/2	-1/2	0	1	35

- (i) Do you agree that this is the best production strategy? If not, improve the solution to get the best production strategy and indicate how much quantity of each product should be produced.

- (ii) Are all the three products produced ? If not, why ?
- (iii) Are all the three resources completely used ? If not, how much of which resource is unused ?
- (iv) Can there be an alternative product mix which gives the same total contribution ?
- (v) What are the marginal worth of the resources ?
- (vi) What happens if 15 machine hours are lost due to some mechanical problem ? (12)
- (b) A firm assembles and sells two different types of outboard motor, A and B, using four resources. The production process can be described as follows :

Resources	Capacity per month
Motor unit shop resource	400 Type A or 250 Type B units or any linear combination of the two
Type A gear and drive shop resource	175 Type A units,
Type B gear and drive shop resource	225 Type B units
Final assembly resource	200 Type A units or 350 Type B units or any linear combination of the two

Type A units bring in a profit of ₹ 90 each and type B units ₹ 60 each. What should be the optimum product mix ? Use graphic method. (6)

OR

Obtain the dual of the following L.P problem :

Minimize $Z = 5x_1 + 10x_2 + 15x_3 + 12x_4$

subject to the constraint :

$$2x_1 + 3x_2 + 7x_3 + x_4 \geq 50$$

$$x_1 + x_2 + x_4 = 45$$

$$-x_1 + 4x_2 + 5x_3 \leq 30$$

$$x_1, x_2, x_3 \geq 0$$

$$x_4 \text{ unrestricted in sign.} \quad (6)$$

3. (a) A firm acquires its annual requirements in 5 orders of 1000 units each at present. If per order cost is ₹ 40, irrespective of the order size, holding cost is 10% of the average rupee inventory and the cost per unit is ₹ 100, find Economic Order Quantity (EOQ) using calculus. Also find annual saving by switching to EOQ system. (6)

OR

The Demand and Total Cost functions of a monopolist are $p = 20 - 4x$ and $C = 4x + 2$ respectively. If the Government imposes tax @ 20% of sales, determine the total tax revenue, that the government will be able to collect. (6)

- (b) The demand functions for two commodities x_1 and x_2 in terms of their respective prices p_1 and p_2 are given by $x_1 = p_1^{-a_1} e^{b_1 p_2 + c_1}$ and $x_2 = p_2^{-a_2} e^{b_2 p_1 + c_2}$, where a_1, a_2, b_1, b_2 and c_1, c_2 are constants.

- (i) Determine the conditions so that the commodities are (a) competitive (b) complimentary.
- (ii) Show that the 'direct' partial price elasticities of demand are independent of the prices, while the 'cross' price partial elasticities of demand are dependent upon price of one of the commodities.

(6)

OR

A producer desires to maximize output (Q). His production function is given by $Q = LK$ where L and K are labour and capital respectively. His cost function is given by $C = 4L + 10K$. Using Lagrange multiplier method, find the optimum combination of inputs if the cost constraint is ₹ 200.

(6)

- (c) Find the Consumers' Surplus and Producers' Surplus under pure competition for demand

function $p_d = \frac{8}{x+1} - 2$ and supply function $p_s = \frac{1}{2}(x+3)$, where p is price and x is quantity.

(6)

OR

After an advertisement campaign, the rate of sales of a product is given by $S(t) = 1000e^{-0.5t}$ where t is the time in months. Find :

- (i) Total cumulative sales after 2 months;
 (ii) Sales during the 3rd month, and
 (iii) Total sales as a result of the campaign.

(6)

4. (a) Suppose a firm has a production function $x = [aL^4 + bK^4]^{\frac{1}{2}}$ where $0 < a < 1$, $0 < b < 1$. L and K are labour and capital respectively. Find the marginal product of labour and marginal product of capital and verify Euler's theorem.

(6)

OR

A discriminating monopolist can separate his consumers into 2 distinct markets with the following demand functions:

$$\text{Market 1 : } Q_1 = 16 - 0.2P_1$$

$$\text{Market 2 : } Q_2 = 180 - 2P_2$$

Assume that the monopolists' total cost function takes the form $TC - 20Q - 20 = 0$, where Q is the total output. Obtain the total profit function and determine the prices he would charge in the two markets to maximise the profit. What is the total profit? Do you agree that the price charged in the market with a higher elasticity of demand would be higher? Show by calculus.

(6)

- (b) At $t = 9$, the annual world use of natural gas was 50 trillion cubic feet which is increasing at the rate of 10% continuously.

(i) Find the total consumption of first 10 years.

(ii) How long will it take to use all available gas if in the beginning it was known that world reserves of gas were 2200 trillion cubic feet?

(6)

OR

The elasticity of cost is given by $e_c = \frac{5x}{2(5x+9)}$. Find the total cost function given that the fixed cost is ₹ 18.

5. (a) A money lender charges interest at the rate of five paise per rupee per month, payable in advance. What effective rate of interest does he charge per annum? (5)

OR

If the present value and amount of an ordinary annuity of ₹ 1 per annum for n years are ₹ 8.1109 and ₹ 12.0061 respectively, find the rate of interest and the value of n without consulting the compound interest table. (5)

- (b) A house is sold for ₹ 50,000 down and 10 semi-annual payments of ₹ 5,000 each, the first due 3 years hence. Find the cash price of the house if money is worth 20% per annum compounded semi-annually. (5)

OR

A person borrows ₹ 12000. He pays ₹ 4000 at the end of 6 months and 5000 at the end of one year. What final payment should be made at the end of 2 years to settle the debt if interest rate is 12% compounded continuously? (5)

- (c) A machine depreciates at the rate of 10% per annum for the first two years and then 7% per annum for the next three years, depreciation being calculated on the diminishing value basis. If the value of the machine is ₹ 10,000 initially, find the average rate of depreciation and the depreciated value of the machine at the end of fifth year. (5)

OR

A debt of ₹ 5,000 with interest at 5% compounded semi-annually is to be amortized by equal semi-annual payments of ₹ R , over the next 3 years, the first due in 6 months.

- (i) Find the value of each installment;
(ii) The interest content of the 5th installment; and
(iii) The principal balance remained after 4th installment. (5)